NORMAL-MODE ANALYSIS OF COUPLED-SLOTS WITH AN AXIALLY-MAGNETIZED FERRITE SUBSTRATE

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ABSTRACT

A normal-mode analysis of axially-magnetized coupled ferrite lines (CFL) is presented for the first time. This will permit optimization of propagation characteristics, impedance-matching and placement of ferrite for device applications. Potential applications include a novel distributed planar circulator which does not suffer the high-frequency limitations of the junction circulator.

I. INTRODUCTION

At higher microwave frequencies, the "drop-in" technology used to fabricate the classical junction circulator becomes very demanding and expensive due to the dependency of its diameter on the wavelength. Possibly a more economic solution is to use a completely new type of (4-port) circulator [1, 2, 3] which has no such constraint. This device consists of a pair of longitudinally-magnetized coupled ferrite lines (CFL) in cascade with a $0^o/180^o$ hybrid coupler.

Although the principle of operation of the CFL section has been explained by Mazur and Mrozowski (MM) using the theory of coupled modes [4], satisfactory optimization of the novel circulator has yet to be achieved. The purpose of this study is to discuss, for the first time, an alternative and complementary viewpoint on the behaviour of the CFL: as the superposition of two normal modes of the *magnetized* structure.

Motivations for proposing this method are as follows. In principle at least, this normal-mode approach should give better accuracy as no theoretical assumptions (such as *weak coupling* in the coupled-mode method [5]) are made. Moreover, optimum positioning of the ferrite layer/slab/rod is best judged from the circularly/elliptically polarized normal-mode vector field plots, and for proper matching at the discontinuities the normal-mode impedances are required. This method, therefore, *must* be considered if satisfactory optimization of the CFL is to be achieved.

II. THEORY

From Coupled Modes to Normal Modes

MM have found that the even- and odd-modes of the *unmagnetized* CFL become coupled when longitudinal magnetization is applied to the ferrite [4]. It can be shown that two dominant modes should result from this. Let $V_{line,mode}(z)$ represent the voltage on each line for each of the two *normal* modes of the *magnetized* CFL. Continuing from MM's analysis in [4], it can be shown that the phase difference ϕ_{mode} between lines 1 and 2 for each mode are

$$\begin{aligned}\phi_1 &= \angle V_{11}(z) - \angle V_{21}(z) \\ &= -2\arctan\left(\frac{\Gamma - \Delta\beta}{C}\right) \qquad (1)\\ \phi_2 &= \angle V_{12}(z) - \angle V_{22}(z) \\ &= 2\arctan\left(\frac{\Gamma + \Delta\beta}{C}\right) \qquad (2)\end{aligned}$$

where

$$\Gamma = \sqrt{\Delta\beta^2 + |C|^2} \qquad (3)$$

$$\Delta\beta = \frac{\beta_{even} - \beta_{odd}}{2} \qquad (4)$$

and C is the coupling coefficient caused by the gyrotropy of the ferrite. Its sign depends on whether the magnetization is forward or reverse. Subtracting eqn. 1 from 2, it can be shown that,

$$|\phi_2 - \phi_1| = 180^o \qquad (5)$$

independent of $\Delta\beta$ and C. However, ϕ_1 and ϕ_2 each *does* depend on $\Delta\beta$ and C. For optimum operation of the magnetized CFL, $\beta_{even} = \beta_{odd}$ [4], which results in the normal-mode condition

$$\phi_1 = -90^o, \qquad \phi_2 = +90^o \qquad (6)$$

It is interesting that, unlike isotropic lines, the phase difference between the lines for each mode of the magnetized CFL is frequency dependent.

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Phase Relations of General Coupled Lines

The question might be asked: why $|\phi_2 - \phi_1| = 180^o$?

Consider a general pair of coupled lines which support two dominant modes with propagation constants β_1 and β_2, where the phase difference between the lines for modes 1 and 2 are ϕ_1 and ϕ_2 respectively. Let us define an arbitrary phase difference θ between modes 1 and 2. With $e^{j\omega t}$ understood, the voltage $V_{line,mode}(z)$ with peak voltage $V_{p,line,mode}$, current $I_{line,mode}(z)$ and impedance $Z_{line,mode}$ for each line and mode is given in Table 1.

Applying superposition of voltages and currents to the expressions in Table 1, it can be shown that the average power $P_{line}(z)$ $(= \frac{1}{2}\mathrm{Re}\,[V_{line}(z)I^*_{line}(z)])$ on each line is given by

$$\begin{aligned} P_1(z) &= \frac{1}{2}\left(\frac{V_{p11}^2}{Z_{11}} + \frac{V_{p12}^2}{Z_{12}}\right) \\ &+ \frac{1}{2}V_{p11}V_{p12}\left(\frac{1}{Z_{11}} + \frac{1}{Z_{12}}\right)\cos(\beta_2 z - \beta_1 z + \theta) \quad (7) \\ P_2(z) &= \frac{1}{2}\left(\frac{V_{p21}^2}{Z_{21}} + \frac{V_{p22}^2}{Z_{22}}\right) + \frac{1}{2}V_{p21}V_{p22}\left(\frac{1}{Z_{21}} + \frac{1}{Z_{22}}\right) \\ &\times \cos(\beta_2 z - \beta_1 z + \theta + \phi_2 - \phi_1) \quad (8) \end{aligned}$$

line 1	mode 1	$V_{11}(z) = V_{p11}e^{-j\beta_1 z}$ $I_{11}(z) = \frac{V_{p11}}{Z_{11}}e^{-j\beta_1 z}$
	mode 2	$V_{12}(z) = V_{p12}e^{-j(\beta_2 z+\theta)}$ $I_{12}(z) = \frac{V_{p12}}{Z_{12}}e^{-j(\beta_2 z+\theta)}$
line 2	mode 1	$V_{21}(z) = V_{p21}e^{-j(\beta_1 z+\phi_1)}$ $I_{21}(z) = \frac{V_{p21}}{Z_{21}}e^{-j(\beta_1 z+\phi_1)}$
	mode 2	$V_{22}(z) = V_{p22}e^{-j(\beta_2 z+\theta+\phi_2)}$ $I_{22}(z) = \frac{V_{p22}}{Z_{22}}e^{-j(\beta_2 z+\theta+\phi_2)}$

Table 1: Expressions for voltages and currents for each line and mode.

For conservation of power of bounded modes in lossless lines, $[P_1(z) + P_2(z)]$ must be constant. This imposes **BOTH** of the following conditions:

$$\begin{aligned} V_{p11}V_{p12}\left(\frac{1}{Z_{11}} + \frac{1}{Z_{12}}\right) &= V_{p21}V_{p22}\left(\frac{1}{Z_{21}} + \frac{1}{Z_{22}}\right) \quad (9) \\ |\phi_2 - \phi_1| &= 180^o \quad (10) \end{aligned}$$

The condition $|\phi_2 - \phi_1| = 180^o$ is therefore a *necessary* physical condition. As an example, this condition is obeyed by the even- (0^o) and odd- (180^o) modes of symmetrical isotropic lines.

Phase Relations of Symmetrical CFL

If the lines are symmetrical, for each mode we can make $V_{p11} = V_{p21} = V_{p12} = V_{p22} = V_p$, and $Z_{11} = Z_{21} = Z_{m1}$ and $Z_{12} = Z_{22} = Z_{m2}$, where "m" stands for "*mode*". If initially $P_2(0) = 0$, θ will work out to be zero. With these values and the condition $|\phi_2 - \phi_1| = 180^o$, the total voltage $V_{line}(z)$ can be simplified to

$$\begin{aligned} V_1(z) &= V_p e^{-j\beta_1 z}[1 + e^{-j(\beta_2 z - \beta_1 z)}] \quad (11) \\ V_2(z) &= V_p e^{-j(\beta_1 z + \phi_1)}[1 - e^{-j(\beta_2 z - \beta_1 z)}] \quad (12) \end{aligned}$$

while eqns. 7 and 8 become

$$P_1(z) = \frac{1}{2}V_p^2\left(\frac{1}{Z_{m1}} + \frac{1}{Z_{m2}}\right)[1 + \cos(\beta_2 z - \beta_1 z)] \quad (13)$$

$$P_2(z) = \frac{1}{2}V_p^2\left(\frac{1}{Z_{m1}} + \frac{1}{Z_{m2}}\right)[1 - \cos(\beta_2 z - \beta_1 z)] \quad (14)$$

For correct operation of the CFL, we require that when $P_1(z) = P_2(z)$, either the "even-mode" or the "odd-mode" is seen. When $P_1(z) = P_2(z)$, we obtain

$$(\beta_1 - \beta_2)z = \frac{\pi}{2}(1 + 2n)\ \text{rad}, \qquad n = 0, 1, 2... \quad (15)$$

When $(\beta_1 - \beta_2)z = \frac{\pi}{2}$ rad, then from eqns. 11 and 12 it can be shown that

$$\angle V_1 - \angle V_2 = \phi_1 - 90^o \quad (16)$$

If we have, say, the "odd-mode" at this value of z, then the values of ϕ_1 and ϕ_2 are

$$\phi_1 = -90^o, \qquad \phi_2 = +90^o \quad (17)$$

as in eqn. 6. Another important relation is that the required length z is given by $(\beta_1 - \beta_2)z = \frac{\pi}{2}$ rad, complementary to MM's $Cz = \pi/4$ condition [4].

The graphs of $P_1(z)$, $P_2(z)$ and $[\angle V_1(z) - \angle V_2(z)]$ under these conditions are shown in Fig. 1(a), where it is seen that the phase difference between lines 1 and 2 is always either 0^o or 180^o, so that when they have equal power either the "even" or "odd" mode is seen. The effect of reversing the direction of magnetization of the CFL must be obtained from a numerical field solution, but the same kind of nonreciprocal effects as in [4] should be observed.

III. NUMERICAL RESULTS

The coupled finline/slotline structure analyzed by Mazur [6] (see Fig. 1(b)) was solved for its E-field using a modified version of the finite element method (FEM) in [7]. The structure was solved for both the unmagnetized and magnetized cases. Preliminary numerical results are shown in Figs. 2(a)–(f). The dispersion diagram for the coupled-slots before and after magnetization in Fig. 2(a) confirms that coupling occurs.

In Fig. 2(b), a comparison is made between our values of $\Delta\beta$ (TD) with Mazur's (M) for the unmagnetized structure, where ours is shifted up by ≈ 20 rad/m, with $\Delta\beta = 0$ at 24GHz instead of at 27.3GHz.

This may be attributed to the high sensitivity of the crossover frequency to errors in β_{even} and β_{odd}.

Figs. 2(d)–(f) show the transverse E-field of the first (RHCP) mode of the magnetized structure at 23GHz, at values of z which are $\lambda/8$ apart. The minima of the slots are $\approx \lambda/4$ apart, hence giving $\phi_1 \approx -90^o$.

The values of $\phi_{1,2}$, obtained in this fashion, are plotted in Fig. 2(c). The normal-mode condition $\phi_{1,2} = \pm 90^o$ is seen to occur at $\approx$ 23GHz. This difference from the value of 24GHz in Fig. 2(b) may be caused by errors in the judgment of phase difference from the field plots. Mazur's values of $\phi_{1,2}$ were calculated by substituting his values of $\Delta\beta$ and C from [6] into eqns. 1 and 2. Note that our values of $\phi_{1,2}$, obtained independently from the fields of each mode, confirm eqn. 10.

Lastly, applying eqn. 15 (with $n = 1$) to Fig. 2(a) at 24GHz, the required CFL length will be $\approx$ 79.4mm. This is different from Mazur's 14.4mm at 27.3GHz [6], and further work is in hand.

IV. CONCLUSION

It has been shown that, for any pair of coupled lines, $|\phi_2 - \phi_1| = 180^o$ (eqn. 10). In the case of the magnetized CFL, there is a complementary normal-mode understanding of its behaviour, where the conditions for optimum operation are $\phi_{1,2} = \pm 90^o$ and $(\beta_1 - \beta_2)L = \frac{\pi}{2}(1+2n)$ rad, where L is the length of the CFL section and n is an integer. The circular/elliptical polarization of the two normal modes seem to cause the unusual modal phase difference between lines. Note that the present work may also be understood from the CPW point of view.

Further work would involve determining the impedance of each normal-mode. Once successfully optimized, this distributed circulator would in principle be compatible with MMICs and therefore would represent a potential breakthrough in nonreciprocal component design.

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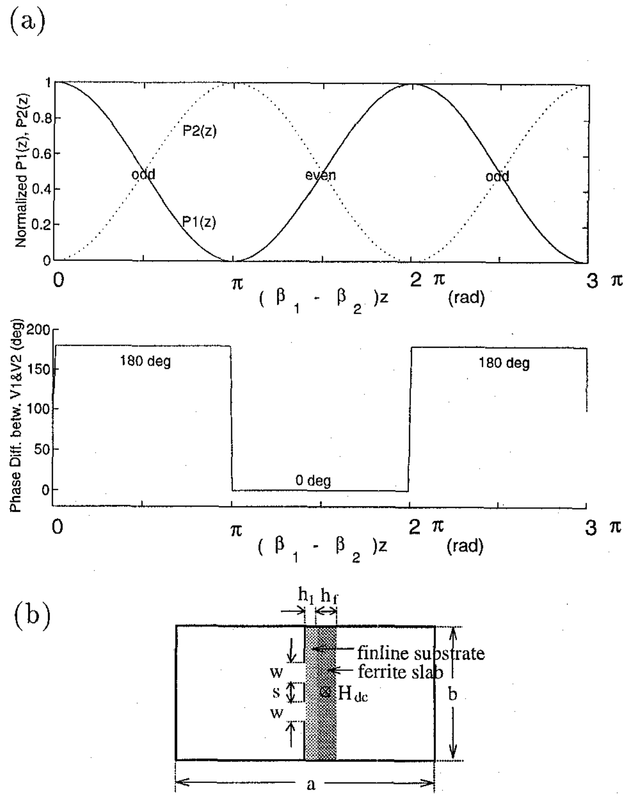
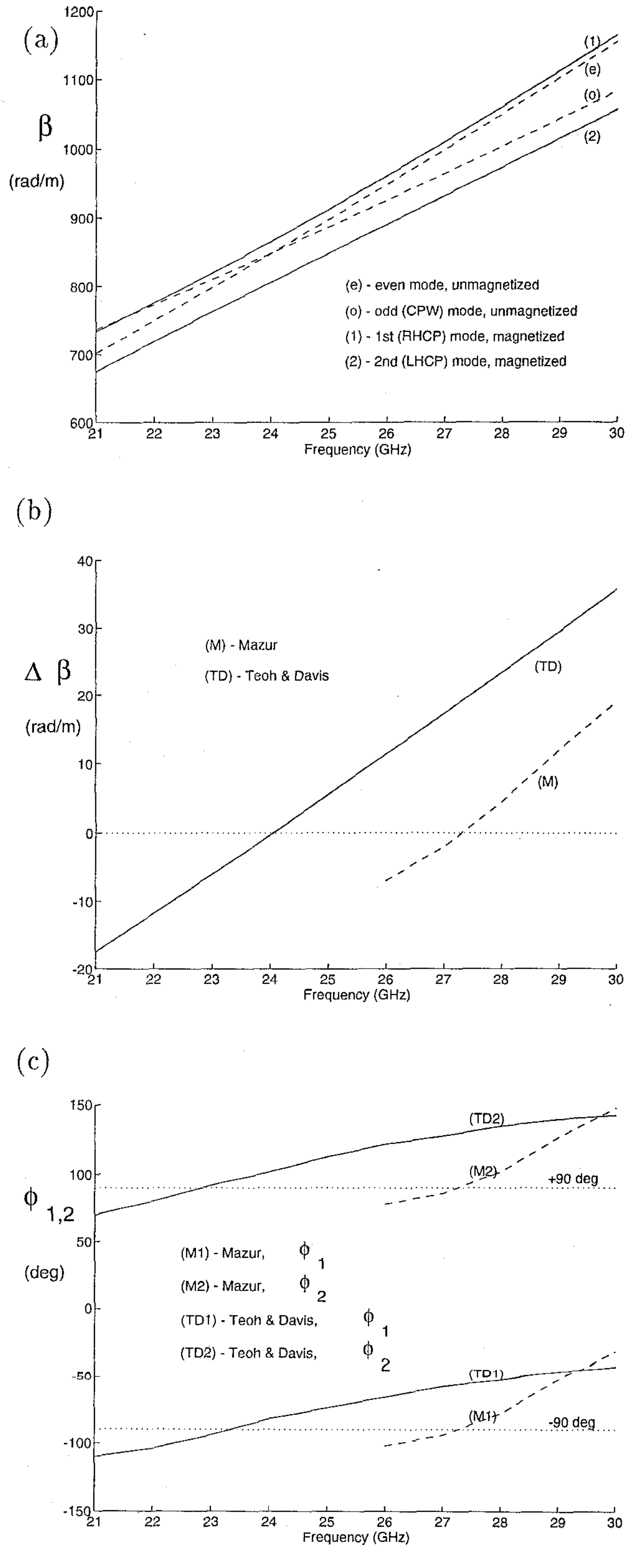


Figure 1: (a) Normalized powers $P_1(z)$, $P_2(z)$ and phase difference $[\angle V_1(z) - \angle V_2(z)]$ for $\phi_1 = -90^o$, $\phi_2 = +90^o$. (b) Cross-section of CFL section. Dimensions in millimeters: $a = 7.2$, $b = 3.4$, $w = s = 0.5$, $h_1 = 0.127$, $h_f = 0.5$. The permittivities are: finline substrate $\epsilon_d = 2.22$, ferrite slab $\epsilon_f = 13.5$, and saturation magnetization $M_s = 340$ kA/m. Ferrite is just saturated ($H_i = 0$) (data from [6]).



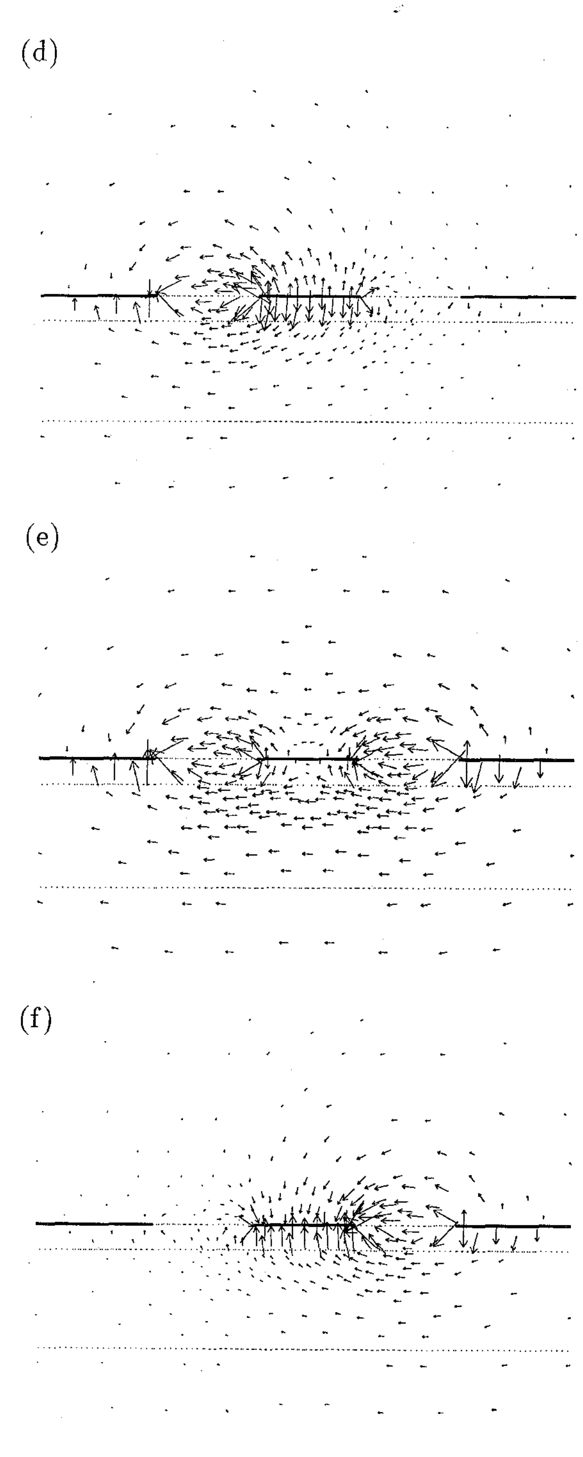


Figure 2: Solutions for coupled-slotlines:
(a) Propagation constants, with and without magnetization
(b) $\Delta\beta$ (see eqn. 4), without magnetization
(c) $\phi_{1,2}$, with magnetization
(d)–(f) E-field plots of mode 1, $\lambda/8$ apart, with magnetization. Field vectors below central strip are Right-Hand Circ. Polarized (RHCP).